A Gateway to All Post Graduate Courses

An MHRD Project under its National Mission on Education throught ICT (NME-ICT)

Subject: Physics

Production of Courseware

-Content for Post Graduate Courses

Paper No. : Electromagnetic Theory

Module : Electromagnetic Waves - I

Development Team

Principal Investigator	Prof. Vinay Gupta, Department of Physics and Astrophysics, University of Delhi, Delhi
Paper Coordinator	Prof. V. K. Gupta, Department of Physics and Astrophysics,University of Delhi, Delhi
Content Writer	Prof. V. K. Gupta, Department of Physics and Astrophysics,University of Delhi, Delhi
Content Reviewer	Prof. Pankaj Sharan, Physics department from JMI, New Delhi

Description of Module	
Subject Name	Physics
Paper Name	Electromagnetic Theory
Module Name/Title	Electromagnetic Waves - I
Module Id	M7

Contents

Learning Objectives:

From this module students may get to know about the following:

1. The wave-like solution of Maxwell's equations in free space away from charges.
2. The propagation of electromagnetic waves in nondispersive medium and how the velocity of the waves depends on the permittivity and permeability of the medium.
3. The propagation of electromagnetic waves in a dispersive medium, a medium in which the permittivity of the medium is a function of frequency of the electromagnetic waves.
4. Electromagnetic waves being transverse, there is polarization of the electromagnetic waves which is expressed in terms of the direction of the electric field in the plane normal to the direction of propagation.
5. The Stokes parameters in terms of which the state of polarization of the wave can be expressed.
6. They come to know briefly about the importance of polarization in astronomical studies.

7.I Introduction

The debate over the nature of light dates back to the 17th century, when Christiaan Huygens and Isaac Newton proposed competing theories of light: In 1630, René Descartes popularized the wave description in his treatise on light, showing that the behavior of light could be re-created by modeling wave-like disturbances in a universal medium. Beginning in 1670 and progressing over three decades, Newton developed and championed his corpuscular hypothesis, arguing that the perfectly straight lines of reflection demonstrated light's particle nature. Around the same time, Newton's contemporaries Robert Hooke and Huygens—and later Fresnel—mathematically refined the wave viewpoint, showing that if light traveled at different speeds in different media (such as water and air), refraction could be easily explained as the medium-dependent propagation of light waves. The resulting Huygens–Fresnel principle, subsequently supported by Thomas Young's 1803 discovery of double-slit interference, was the beginning of the end for the particle light camp. The final blow against corpuscular theory came when James Clerk Maxwell realised that he could combine four simple equations, which had been previously discovered, along with a slight modification to describe self-propagating waves of oscillating electric and magnetic fields. When the propagation speed of these electromagnetic waves was calculated, it turned out to be just the speed of light. It quickly became apparent that visible light, ultraviolet light, and infrared light were all electromagnetic waves of differing frequency. The wave theory had prevailed (or at least it seemed to, till the advent of quantum theory).

These electromagnetic waves are produced by sources which may consist of a moving particle or a localized oscillating current. For the moment however we are interested in the propagation of these waves in vacuum or various kinds of media, once they have been produced. So for the time being we work in a region of space away from the sources. To solve for the Maxwell equations in the presence of sources requires the introduction of the auxiliary quantities – the scalar and the vector potentials. However in the source-free region Maxwell's equations can be solved and the existence of wave-like solutions can be demonstrated without the need to introduce potentials.

7.2 Solution of Maxwell's Equations

7.2.1 Nondispersive Medium

The Maxwell equations in a medium are

$$\vec{\nabla}.\vec{D} = \rho/\varepsilon \ , \qquad \vec{\nabla}\times\vec{H} = \mu_0\vec{J} + \partial\vec{D}/\partial t \qquad (1)$$

$$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0\,, \qquad \vec{\nabla}.\vec{B} = 0 \qquad (2)$$

Here $\vec{E}(\vec{x},t)$ and $\vec{B}(\vec{x},t)$ are the electric field and magnetic induction respectively at the point $\vec{x}$ at time t. $\vec{D}$ and $\vec{H}$ are the auxiliary fields which are introduced to take the effect of the medium into account – they are the macroscopic fields. ρ and $\vec{J}$ are respectively the charge and current densities at ($\vec{x}$,t). These equations have to be supplemented by material equations which express the derived quantities, $\vec{D}$ and $\vec{H}$ in terms of the primary fields $\vec{E}$ and $\vec{B}$. For a continuous and homogenous medium these relations take the form

$$\vec{D} = \varepsilon\vec{E}, \qquad \vec{B} = \mu\vec{H} \,. \tag{3}$$

The constants ε and μ are called the permitivity and the permiability of the medium respectively. In case we are interested in the study of electromagnetic phenomena in vacuum or where the effect of the medium can be ignored, ε and μ can be replaced by ε_0 and μ_0, the permitivity and the permiability of vacuum, respectively.

In the absence of the sources in an infinite medium the inhomogeneous term are absent and the equations become

$$\vec{\nabla}.\vec{D} = 0 \qquad \vec{\nabla}\times\vec{H} = \partial\vec{D}/\partial t \tag{4}$$

$$\vec{\nabla}.\vec{B} = 0 \qquad \vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \,, \tag{5}$$

Further on using the auxiliary relation, they can be recast in the form

$$\vec{\nabla}.\vec{E} = 0 \tag{6}$$

$$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \tag{7}$$

$$\vec{\nabla}.\vec{B} = 0 \tag{8}$$

$$\vec{\nabla}\times\vec{B}-\mu\varepsilon\frac{\partial\vec{E}}{\partial t}=0 \quad (9)$$

To solve these equations, take the curl of the last equation (9):

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B})-\mu\varepsilon\frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t}=0$$

On using equations (7), (8) and the vector identity

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B})=\vec{\nabla}(\vec{\nabla}.\vec{B})-\nabla^2\vec{B}$$

we get

$$\nabla^2\vec{B}-\frac{1}{v^2}\frac{\partial^2\vec{B}}{\partial t^2}=0\,. \quad (10)$$

Similarly on taking the curl of the second equation (7) and using equations (6) and (9), we get

$$\nabla^2\vec{E}-\frac{1}{v^2}\frac{\partial^2\vec{E}}{\partial t^2}=0\,. \quad (11)$$

In these equations

$$v=\frac{1}{\sqrt{\varepsilon\mu}}\,. \quad (12)$$

Thus both $\vec{E}$ and $\vec{B}$ satisfy the wave equation

$$\nabla^2 u - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0 \qquad (13)$$

The parameter με has the dimensions of (velocity)$^{-2}$ and so v has the dimensions of velocity and represents the velocity of propagation of the wave. Like any second order linear differential equation, wave equation has two linearly independent solutions. The simplest of these are the well-known plane-wave solutions:

$$u = e^{\pm i\vec{k}.\vec{x} - i\omega t}. \qquad (14)$$

ω is the frequency and $\vec{k}$ is the wave vector. The frequency ω and the magnitude of the wave vector k are related by

$$k = \frac{\omega}{v} = \sqrt{\mu\varepsilon}\,\omega = \sqrt{\kappa\kappa_m}\,\frac{\omega}{c}, \qquad (15)$$

where $\kappa = \varepsilon / \varepsilon_0$ is the dielectric constant of the medium and $\kappa_m = \mu / \mu_0$ its relative permittivity. If we restrict ourselves to only one dimension for the time being, say the x-direction, the fundamental solution is the linear combination of the two solutions above:

$$\begin{aligned} u(x,t) &= Ae^{ikx - i\omega t} + Be^{-ikx + i\omega t} \\ &= Ae^{ik(x - vt)} + Be^{-ik(x+vt)} \end{aligned} \qquad (16)$$

If the medium is nondispersive, that is, if μ and ε are independent of the frequency, then v is not a function of k, then the general solution can be obtained by taking the Fourier transform of the above solution. In fact we can verify by direct substitution that the general solution of the one-dimensional wave equation is

$$u(x,t) = f(x - vt) + g(x + vt), \qquad (17)$$

where f and g are arbitrary functions. The first term represent a wave traveling to the right and the second to the left. The velocity of propagation is v, which is called the phase velocity of the wave.

7.2.2 Dispersive medium

If the medium is dispersive, that is, if μ and ε depend on the frequency, then v is a function of k. The above discussion needs some modification. Let us take the Fourier transform of the Maxwell's equations before combining them together to obtain the wave equation. So let

$$\begin{aligned}\vec{E}(\vec{x},t) &= \int_{-\infty}^{\infty} \vec{E}(\vec{x},\omega)e^{-i\omega t}d\omega \\ \vec{B}(\vec{x},t) &= \int_{-\infty}^{\infty} \vec{B}(\vec{x},\omega)e^{-i\omega t}d\omega\end{aligned} \qquad (18)$$

The partial derivative with time can be replaced by the factor (-iω). From the inverse Fourier Transform theorem

$$\begin{aligned}\vec{E}(\vec{x},w) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \vec{E}(\vec{x},t)e^{i\omega t}dt \\ \vec{B}(\vec{x},w) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \vec{B}(\vec{x},t)e^{i\omega t}dt\end{aligned} \qquad (19)$$

Substituting equation (12) into the Maxwell's equations (6) – (9) and equating the integrands we obtain

$$\vec{\nabla}.\vec{E} = 0 \qquad (20)$$

$$\vec{\nabla}\times\vec{E} - i\omega\vec{B} = 0 \qquad (21)$$

$$\vec{\nabla}.\vec{B} = 0 \quad (22)$$

$$\vec{\nabla} \times \vec{B} + i\omega\mu\varepsilon\vec{E} = 0 \quad (23)$$

Following the same procedure as we did earlier, viz., taking the curl of second or fourth equation and making use of others, we now obtain

$$\begin{aligned} \nabla^2\vec{E} + \frac{\omega^2}{v^2}\vec{E} &= 0 \\ \nabla^2\vec{B} + \frac{\omega^2}{v^2}\vec{B} &= 0 \end{aligned} \quad (24)$$

The electric and magnetic fields are now both solutions of the Helmholtz wave equation

$$\nabla^2 u + \frac{\omega^2}{v^2}u = 0 \quad (25)$$

This equation has two linearly independent solutions: $\vec{u}(\vec{x}) = e^{\pm i\vec{k}.\vec{x}}, k = \sqrt{\mu\varepsilon}\,\omega = \frac{\omega}{v}$. This means k is still given by the same equation (15), and equation (16) gives the plane wave solution for each frequency. Only when we reconstitute the solution $u(\vec{x},t)$ by using equation (18) that dispersion produces modifications. The general solution (17) no longer holds.

7.2.2.1 The Plane Wave Solution

The basic plane wave (14) with k given by (15) satisfies the scalar wave equation (25). But we are looking for a solution of the Maxwell equations, so that our solution must satisfy the Maxwell equations as well. So let us assume that each component of the electric field $\vec{E}$ and magnetic induction $\vec{B}$ satisfies the scalar wave equation and write these fields in the form

$$\vec{E}(\vec{x},t) = \vec{E}_0 e^{ik\hat{n}.\vec{x} - i\omega t}$$
$$\vec{B}(\vec{x},t) = \vec{B}_0 e^{ik\hat{n}.\vec{x} - i\omega t} \qquad (26)$$

These fields as written above are complex objects. The physical fields are of course real. We make the usual convention that physical fields are obtained from the real parts of the complex quantities. The constants $\vec{E}_0, \vec{B}_0$ and $\hat{n}$ are vectors that represent the amplitude of the electric field, magnetic induction and a unit vector along the direction of propagation, respectively. Since each component of $\vec{E}$ and $\vec{B}$ satisfies the scalar wave equation (25), substituting the above representation (26) into the wave equation (25), we obtain the condition

$$k^2 \hat{n}.\hat{n} = \mu\varepsilon\omega^2 = \frac{\omega^2}{v^2}$$

To recover the relation between the wave vector k and frequency ω, equation (15), we must have $\hat{n}.\hat{n} = 1$. (The possibility that $\hat{n}$ is complex cannot be discounted.)

With these requirements, the wave equation is satisfied. Further the solutions must satisfy the Maxwell's equations. From the divergence equations (6) and (8), we obtain

$$\hat{n}.\vec{E}_0 = 0 \qquad (27)$$

$$\hat{n}.\vec{B}_0 = 0 \qquad (28)$$

This means that electric and magnetic fields are both perpendicular to $\hat{n}$, the direction of propagation. As we all know, such a wave in which the disturbance is normal to the direction of propagation, is called transverse wave. The two curl equations (7) and (9) of course must also be satisfied. Substituting the proposed solution (26) into either of the curl equations leads to the further condition

$$\vec{B}_0 = \sqrt{\mu\varepsilon}\hat{n} \times \vec{E}_0 .$$

Thus the electric and magnetic fields are mutually perpendicular as well. If $\hat{n}$ is real, the fields $\vec{E}$ and $\vec{B}$ have the same phase. Since $\vec{E}$, $\vec{B}$ and $\hat{n}$ are all mutually perpendicular, it is useful to introduce a set of right-handed mutually orthogonal unit vectors $(\hat{\varepsilon}_1, \hat{\varepsilon}_2, \hat{n})$ (for example, along the x-, y- and z- directions). If $\vec{E}$ is along $\hat{\varepsilon}_1$ then $\vec{B}$ is along $\hat{\varepsilon}_2$, and if $\vec{E}$ is along $\hat{\varepsilon}_2$ then $\vec{B}$ is along $-\hat{\varepsilon}_1$.Thus we have two linearly independent solutions

$$\vec{E}_0 = E_0 \hat{\varepsilon}_1, \qquad \vec{B}_0 = \sqrt{\mu\varepsilon} E_0 \hat{\varepsilon}_2 \tag{29a}$$

$$\vec{E}_0 = E'_0 \hat{\varepsilon}_2, \qquad \vec{B}_0 = -\sqrt{\mu\varepsilon} E'_0 \hat{\varepsilon}_1 \tag{29b}$$

The waves described by equations (26) and (29a) or (29b) are transverse waves propagating in the direction of $\hat{n}$. The flow of energy is represented by the Poynting vector $\vec{S} = \vec{E} \times \vec{H} = \vec{E} \times \vec{B} / \mu$. Since the physical fields are real parts of the complex fields that we have introduced, they have a time dependence given by $\cos(\omega t)$ or $\sin(\omega t)$, and hence the energy flow across any surface varies rapidly in time with frequency ω. What interests us therefore is the time averaged energy flow across any surface and this is given by

$$< \vec{S} > = \frac{1}{2} \vec{E} \times \vec{B}^* / \mu$$

On using equations (16) and (17) we obtain

$$< \vec{S} > = \frac{1}{2} \sqrt{\frac{\varepsilon}{\mu}} |E_0|^2 \hat{n} \tag{30}$$

Since the energy density in the electromagnetic field is $u = \frac{1}{2}(\varepsilon |\vec{E}|^2 + \frac{1}{\mu}|\vec{B}|^2)$, the time averaged energy density in the wave is

$$< u >= \frac{1}{4}(\varepsilon \vec{E}.\vec{E}* + \frac{1}{\mu}\vec{B}.\vec{B}*) .$$

This gives

$$< u >= \frac{\varepsilon}{2}|E_0|^2 \qquad (31)$$

Equation (30) represents energy flow per unit area per unit time while equation (31) represents energy density per unit volume. The ratio of the two, $|\vec{S}|/u = 1/\sqrt{\mu\varepsilon}$ is the speed of the energy flow, as expected.

7.3 Polarization [See Figures – Griffiths Figure 9.8a and b]

The plane wave described by equations (26) and (29a) is a wave with its electric field vector always pointing in the direction $\hat{\varepsilon}_1$. Such a wave is said to be linearly polarized with polarization vector $\hat{\varepsilon}_1$. Similarly plane wave described by equations (26) and (29b) is linearly polarized with polarization vector $\hat{\varepsilon}_2$. The two waves are linearly independent of each other. Thus the two waves

$$\begin{aligned}\vec{E}_1 &= \hat{\varepsilon}_1 E_1 e^{i\vec{k}.\vec{x} - i\omega t} \\ \vec{E}_2 &= \hat{\varepsilon}_2 E_2 e^{i\vec{k}.\vec{x} - i\omega t}\end{aligned} \qquad (32)$$

with

$$\vec{B}_j = \sqrt{\mu\varepsilon}\,\frac{\vec{k}\times\vec{E}_j}{k}, \qquad j = 1,2 \tag{33}$$

can be combined to give the most general plane wave propagating in the direction of the vector $\vec{k} = k\hat{n}$

$$\vec{E}(\vec{x},t) = (\hat{\varepsilon}_1 E_1 + \hat{\varepsilon}_2 E_2)e^{i\vec{k}.\vec{x}-i\omega t} \tag{34}$$

The amplitudes E_1 and E_2 are complex numbers in order to allow the possibility of a phase difference between waves of different polarization.

If the orientation of the electric vector in an electromagnetic wave changes randomly, it is said to be unpolarized. Natural light is unpolarized.

If E_1 and E_2 have the same phase, equation (34) still represents a linearly polarized wave but with its polarization vector making an angle $\theta = \tan^{-1}(E_2/E_1)$ with $\hat{\varepsilon}_1$ and with a magnitude $E = \sqrt{E_1^{\,2} + E_2^{\,2}}$. **[See Figure- Griffiths 9.8c]**

If E_1 and E_2 have different phases, the wave given by equation (34) is, in general, elliptically polarized. To see it mathematically, consider the wave to be traveling in the z-direction and let us write E_1 and E_2 in terms of magnitude and phase as

$$E_1 = a_1 e^{i\delta_1}, \qquad E_2 = a_2 e^{i\delta_2} \tag{35}$$

Then equation (34) can be written as

$$\vec{E} = \hat{\varepsilon}_1 a_1 e^{i(kz-\omega t+\delta_1)} + \hat{\varepsilon}_2 a_2 e^{i(kz-\omega t+\delta_2)}$$

Since the physical fields are the real parts of the complex quantities,

$$E_x = a_1 \cos(kz - \omega t + \delta_1),$$
$$E_y(\vec{x},t) = a_2 \cos(kz - \omega t + \delta_2)$$

A little manipulation using trigonometric relations leads to

$$\left(\frac{E_x}{a_1}\right)^2 + \left(\frac{E_y}{a_2}\right)^2 - 2\left(\frac{E_x}{a_1}\right)\left(\frac{E_y}{a_2}\right)\cos(\delta_2 - \delta_1) = \sin^2(\delta_2 - \delta_1) \qquad (36)$$

which is the equation of an ellipse. To make the matters more clear, let us first consider the simpler case of circular polarization. In this case E_1 and E_2 have the same magnitude and differ in phase by $\pi/2$. The wave given by equation (34) becomes

$$\vec{E}(\vec{x},t) = E_0(\hat{\varepsilon}_1 \pm i\hat{\varepsilon}_2)e^{i\vec{k}.\vec{x}-i\omega t} \qquad (37)$$

where E_0 is the common (real) amplitude of the two waves and the factor $\pm i$ comes from the phase difference of $\pi/2$ between the two. Specifically, let the direction of propagation of the wave be taken as the positive z-axis, and the unit vectors $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ along the x- and y- directions respectively. Then the actual electric field which is the real part of equation (25) has the components

$$E_x(\vec{x},t) = E_0 \cos(kz - \omega t)$$
$$E_y(\vec{x},t) = \mp E_0 \sin(kz - \omega t) \qquad (38)$$

At a fixed point in space, we see that the electric field (and so of course the magnetic field) is constant in magnitude, but sweeps around in a circle with angular frequency ω. **[See Figure 7.3 from Jackson]** For the upper sign in equation (37) or the second of equation (38) the rotation is counterclockwise when the observer is looking into the oncoming wave. In optics this wave is called left circularly polarized. The lower sign corresponds to clockwise rotation, or right circularly polarized light. In the terminology usually adopted in particle physics, the two are said to posses positive helicity and negative helicity, respectively.

The two circularly polarized waves (37) can be equally well used as a set of basic fields for description of a general state of polarization. For this purpose we introduce the complex orthogonal unit vectors

$$\hat{\varepsilon}_{\pm} = \frac{1}{\sqrt{2}}(\hat{\varepsilon}_1 \pm i\hat{\varepsilon}_2) \qquad (39)$$

with properties

$$\hat{\varepsilon}^*_{\pm}.\hat{\varepsilon}_{\mp} = 0; \qquad \hat{\varepsilon}^*_{\pm}.\hat{\varepsilon}_3 = 0; \qquad \hat{\varepsilon}^*_{\pm}.\hat{\varepsilon}_{\pm} = 1; \qquad (40)$$

Then an alternative representation equivalent to equation (34) is

$$\vec{E}(\vec{x},t) = (\hat{\varepsilon}_+ E_+ + \hat{\varepsilon}_- E_-)e^{i\vec{k}.\vec{x}-i\omega t} \qquad (41)$$

Here E_+ and E_- are complex amplitudes. In general this equation represents an elliptically polarized wave. If the two amplitudes, E_+ and E_-, have the same magnitudes and phases, this represents a linearly polarized wave. If the amplitudes are different but the phases are same then it represents an elliptically polarized wave with the principle axes of the ellipse in the directions $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$. The ratio of the semi-major to semi-minor axis is $\left|\frac{1+r}{1-r}\right|$, where $r = \frac{E_-}{E_+}$. If the amplitudes have a phase difference between them, say, $\frac{E_-}{E_+} = re^{i\alpha}$, then we can easily check that the axes of the ellipse traced out by the $\vec{E}$ vector is rotated by an amount ($\alpha/2$). **[See Figure 7.4 from Jackson]**

7.3.1 Stokes Parameters

The state of polarization is completely determined in terms of four parameters: the two amplitudes and the magnitude and sign of the phase shift, $\delta_2 - \delta_1$. Because the phase difference is hard to measure directly, an alternative description in terms of Stoke's parameters is very useful. In terms of the electric field vector, the stokes parameters can be motivated by observing that for a wave propagating in the z-direction, the scalar products,

$$\hat{\varepsilon}_1.\vec{E}, \quad \hat{\varepsilon}_2.\vec{E}, \quad \hat{\varepsilon}_+^*.\vec{E}, \quad \hat{\varepsilon}_-^*.\vec{E}$$

are the amplitudes of radiation with linear polarization in the x-direction, linear polarization in the y-direction, positive helicity and negative helicity, respectively. The squares of these amplitudes give a measure of the intensity of each type of polarization. The phase information is obtained from the cross products. Just as we have written equation (37) in terms of real amplitude and phase [equation (38)], let us write equation (41) also as

$$E_+ = a_+ e^{i\delta_+}, \quad E_- = a_- e^{i\delta_-}$$

In terms of the linear polarization basis ($\hat{\varepsilon}_1, \hat{\varepsilon}_2$), the Stokes parameters are

$$S_0 = \left|\hat{\varepsilon}_1.\vec{E}\right|^2 + \left|\hat{\varepsilon}_2.\vec{E}\right|^2 = a_1^2 + a_2^2$$

$$S_1 = \left|\hat{\varepsilon}_1.\vec{E}\right|^2 - \left|\hat{\varepsilon}_2.\vec{E}\right|^2 = a_1^2 - a_2^2$$

$$S_2 = 2\,\mathrm{Re}[(\hat{\varepsilon}_1.\vec{E})^*(\hat{\varepsilon}_2.\vec{E})] = 2a_1 a_2 \cos(\delta_2 - \delta_1)$$

$$S_3 = 2\,\mathrm{Im}[(\hat{\varepsilon}_1.\vec{E})^*(\hat{\varepsilon}_2.\vec{E})] = 2a_1 a_2 \sin(\delta_2 - \delta_1)$$

The parameter S_0 measure the relative intensity of the wave. The parameter S_1 gives the preponderance of x-linear polarization over y-linear polarization while S_2 and S_3 give phase information. The four stokes parameters are not independent since they depend on only three quantities: a_1, a_2 and $\delta_2 - \delta_1$. They satisfy the relation

$$S_0^{\,2} = S_1^{\,2} + S_2^{\,2} + S_3^{\,2}$$

We usually normalize the intensity so that the intensity of the incoming light is 1.As an example, let the incoming beam of light be unpolarized. In that case

$$S_0 = 1; \;\; S_1 = S_2 = S_3 = 0 \,.$$

7.3.2 Polarization in Astronomy

Light Polarization is an important phenomenon in astronomy. The polarization of starlight was first observed by the astronomers William Hiltner and John S. Hall in 1949. Subsequently, Jesse Greenstein and Leverett Davis, Jr. developed theories allowing the use of polarization data to trace interstellar magnetic fields. Though the integrated thermal radiation of stars is not usually appreciably polarized at source, scattering by interstellar dust can impose polarization on starlight over long distances. Both circular and linear polarization of light from the Sun has been measured. Circular polarization is mainly due to transmission and absorption effects in strongly magnetic regions of the Sun's surface. Linear polarization in spectral lines is usually created by anisotropic scattering of photons on atoms and ions.

The polarization of the cosmic microwave background (CMB) is also being used to study the physics of the very early universe. CMB exhibits 2 components of polarization: B-mode (divergence-free like magnetic field) and E-mode (curl-free gradient-only like electric field) polarization. The BICEP2 telescope located at the South Pole helped in the detection of B-mode polarization in the CMB. This may prove the existence of Gravitational Waves in our ever inflating universe but confirmation is needed.

It has been suggested that astronomical sources of polarized light caused the chirality found in biological molecules on Earth.

Summary

1. In this module we have first given to the student the wave-like solution of Maxwell's equations in free space away from charges.
2. We then discuss the propagation of electromagnetic waves in nondispersive medium and find how the velocity of the waves depends on the permittivity and permeability of the medium.

3. Next we discuss the propagation of electromagnetic waves in a dispersive medium. A dispersive medium in which the permittivity of the medium depends on the frequency of the electromagnetic waves.
4. Next the question of polarization of the waves is discussed. The polarization is defined in terms of the direction of the electric field in the wave.
5. The Stokes parameters, in terms of which the state of polarization of the wave can be expressed, are defined.
6. Finally the importance of polarization in astronomical studies is briefly described.